How does energy store in a capacitor? Derive an expression for energy.

(Lhr. 2017; Mtn. 2016, 2017; Rwp. 2010; Fsd. 2017)

Capacitor is a device to ***store charge***. it is also possible to think of a capacitor as a device for ***storing electrical energy***. When we ***charge a capacitor***, ***work has to be done***. This ***work*** done while charging capacitor ***increases the electrical potential energy of the capacitor***. When we charge a capacitor the potential difference between plates ***increases*** it means a ***large amount of work is*** needed to bring up next increment of charge.

Initially when the capacitor is ***uncharged***. The ***potential difference*** between plates is ***zero*** and ***finally*** it becomes ***V*** when charge ‘q’ is deposited on each plate.

Thus average potential difference = $\Delta V = \dfrac{0+V}{2} = \dfrac{V}{2}$

Since, Energy $= \text{P.E} = \Delta U$

$$= q\,\Delta V$$

$$\therefore \quad = q\left(\frac{V}{2}\right) \qquad \left[\because \Delta V = \left(\frac{V}{2}\right)\right]$$

$$= \frac{1}{2}qV$$

For a capacitor:

$$q = CV$$

$$\therefore \quad \text{Energy} = \frac{1}{2}(CV)V = \frac{1}{2}CV^2$$

$$\boxed{\text{Energy} = \frac{1}{2}CV^2} \qquad \text{———} (1)$$

Energy Stored in Term of Electric Field:

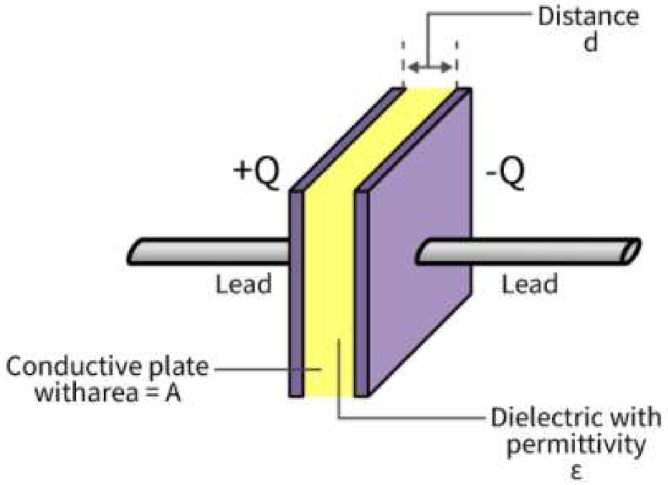
Since, $$C = \frac{A\in_o\in_r}{d} \quad \text{and} \quad V = Ed$$

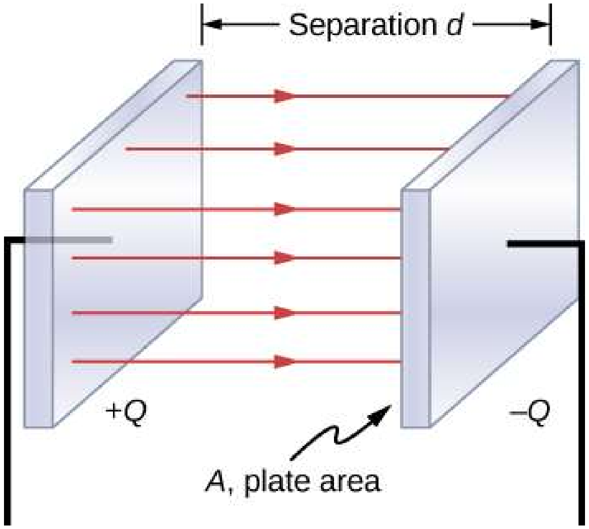
∴ $$\text{Energy} = \frac{1}{2}\left(\frac{A\in_o\in_r}{d}\right)(Ed)^2$$

$$= \frac{1}{2}\frac{A\in_o\in_r}{d}E^2d^2 = \frac{1}{2}A\in_o\in_r E^2d$$

Where, (Ad) = Volume between the plates = V

∴ $$\text{Energy} = \frac{1}{2}\in_o\in_r E^2V$$





Energy Density:

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$\therefore \quad \text{Energy density} = \frac{\frac{1}{2}\in_o \in_r E^2 V}{V} = \frac{1}{2}\in_o \in_r E^2$$

$$\boxed{\text{Energy density} = \frac{1}{2}\in_o \in_r E^2}$$

This equation is valid for any electric field strength.

Problem 12.12:

The electronic flash attachment for a camera contains a capacitor for storing the energy used to produce the flash. In one such unit, the potential difference between the plates of a 750 μF capacitor is 330 V. Determine the energy that is used to produce the flash. **[Ans: 40.8 J]**

Data: *(Lhr. 2015 G-II)*

Potential difference = V = 330 volts

Capacitance = C = 750 μF = 750×10^{-6} F

To Determine:

Energy stored = E = ?

Calculations:

We know that:

$$E = \frac{1}{2}CV^2$$

Putting values, we get:

$$E = \frac{1}{2}(750 \times 10^{-6})(330)^2$$

$$= \frac{1}{2} \times 750 \times 10^{-6} \times 108900$$

$$\boxed{E = 40.8\ \text{J}}$$

Problem 12.13:

A capacitor has a capacitance of 2.5×10^{-8} F. In the charging process, electrons are removed form one plate and placed on the other one. When the potential difference between the plates is 450 V, how many electrons have been transferred? ($e = 1.60 \times 10^{-16}$ C) [Ans: 7.0×10^{13} electrons]

(Lhr. 2012)

Data:

Capacitance = $C = 2.5 \times 10^{-8}$ F

Potential difference = $V = 450$ V

Charge on electrons = $e = 1.6 \times 10^{-19}$ C

To Determine:

No. of electrons = n = ?

Information

No. of electrons in 1 coulomb?

$$Q = ne$$

$$n = \frac{Q}{e} = \frac{1}{1.6 \times 10^{-19}}$$

$$n = 6.25 \times 10^{18} \text{ electrons}$$

Calculations:

We know that:

$$Q = CV$$

But

$$Q = ne$$

$$\therefore \quad ne = CV$$

$$n = \frac{CV}{e}$$

Putting values, we get:

$$n = \frac{(2.5 \times 10^{-8})(450)}{1.6 \times 10^{-19}}$$

$$= \frac{2.5 \times 450}{1.6} \times 10^{-8+19} = \frac{1125}{1.6} \times 10^{11}$$

$$= 703.2 \times 10^{11}$$

$$\boxed{n = 7.0 \times 10^{13} \text{ electrons}}$$